HW 046203 Wet 1

Question 3

1. As shown in class we can linearize the inverted pendulum on a cart around :

.

When approximating with first order:

with

We fill the code accordingly.

1. We choose the following cost parameters to stabilize the pendulum with initial position :

A screenshot of a computer

Description automatically generated with low confidence

We recall that Q matrix is applied to the state vector and R is applied to the action.

We consider in the cost function:

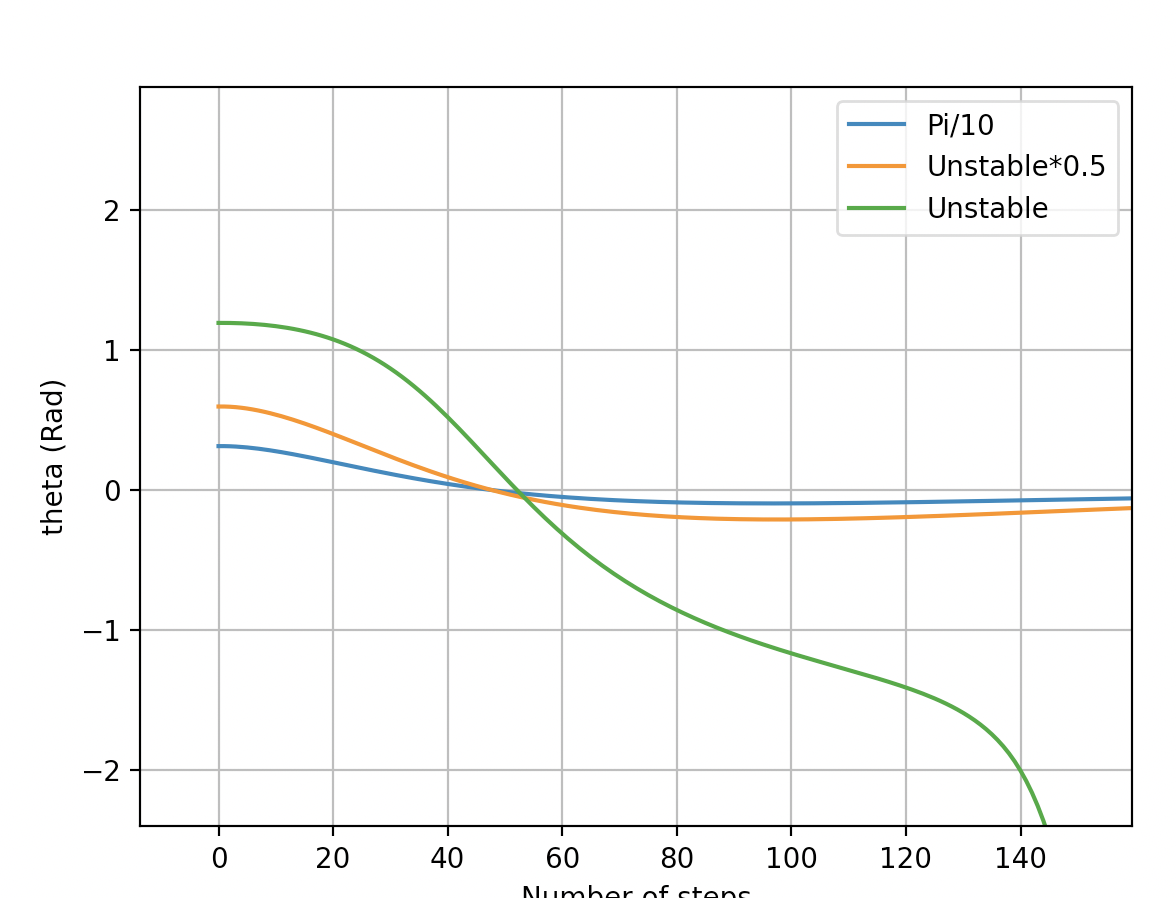
* Cart position, we don’t want the cart to get away from its start point too much,
* Cart velocity, we want the cart to be relatively stable and not increase velocity,
* Pendulum angle, of course we want to have small pendulum angle,
* Angular velocity since we want stability in the angle also.
* The force of the action, since applying the force on pendulum may have a cost in real-life and we don’t want to over utilize it.

1. We find that under these cost parameters.

The plots of theta over time are as follows:

Chart, line chart

Description automatically generated



As we can see, for , the pendulum converges to stable upright position whereas for , the pendulum keeps diverging.

The difference is due to the limitations of the LQR model, it approximates the pendulum dynamics as a first order-approximation around . Then, when , the approximation doesn’t hold as well as for small angles.

1. The plots using the feedforward control law are as follows:

Chart, line chart

Description automatically generated

This LQR control law doesn’t converge to the upright pendulum position even for small initial angles like .

We conclude easily that the feedback LQR control is better than the feedforward LQR control. In feedforward control law, the actions are computed only based on predicted states of the pendulum. However, the pendulum dynamics are approximated (with first order approx..) during LQR computation. This creates a difference with the real state computed by the simulator (using full dynamics) and the predicted actions don’t fit at all to the actual states.

1. Using our previous cost parameters, and limiting the maximal force to 4, we find that the pendulum converges to upright position when initial theta is .

Also, we observe with some trials that .

However, it is obvious that if we increase the action cost parameter R, then the LQR control law will likely stabilize with larger initial theta (increasing to makes the initial angle stable) since the larger R, the smaller forces the LQR controller applies.

Here are the plots with :

Chart, line chart

Description automatically generated